The Adequacy of Language for Finite Domains of Reference

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Abstract

A formal system is constructed which generates a finite set of statements that correlate 1-to-1 with the facts of a finite domain. Each formal statement transcribes directly to English, so the adequacy of language in general is entailed by the adequacy of the formal system. The crux of the argument involves giving precision to the phrase "finite domain of reference." It is assumed that any such domain consists of a finite number of individuals connected to one another by a finite number of relations. A domain of three individuals is examined in detail, which serves to show that any relation is constructible from a pair-wise relation. This in turn implies that the realm of finite mathematical structure can be systematically generated by an algorithm. An important domain of reference for the formalism is obtained by interpreting the primitive relation as "time order." We then see what Russell and Whitehead's "eventism" looks like when restricted to a finite domain.

"Logic gives boldness." Thus spake Whitehead. And logic, for Whitehead, is largely a matter of formal notation, judging by the years he devoted to *Principia Mathematica* His partner in that project, Bertrand Russell, showed similar commitment to the expression of ideas in formal notation. Such formalism involves putting marks to paper and saying "There—that expresses what I want to say—that's my idea." The idea is now freestanding—naked in the public square and open to criticism. There is an incubation period for ideas, to be sure, preceding the act of formulation, but this may turn out to be a "false pregnancy." It is best to accept the challenge once put to me by a teacher: "If you can't say it, you don't understand it."

The most important idea in the formal construction of mathematics is that of a "relation" and its "relata." The three volumes of Principia that were completed deal with the simplest form of relation, which is called "dyadic," or "2-termed." The next volume was intended to deal with "3-termed relations," and higher-termed relations, by which geometry would be formalized. At that point, Whitehead's interest was diverted from pure mathematics to applied geometry—namely, the geometry of space-time—which led to "eventism." But what *are* relations? And what is meant by their "termed-ness?" Let's attack it formally.

We can formalize the simplest possible statement as follows: R(a,b). This states that a and b are related by R, a dyadic relation. But isn't there a predicate form of statement, "P(a)," that is simpler yet? We have a property or predicate "P" applied to a lone individual, "a." "P(a)" states that individual a has the property P. The keyword in the last sentence is the word "has," which belies the implicit relation between an individual and its property. Juxtaposition itself is being employed in place of an explicit relation symbol. Resistance to the very idea of "relation" fostered the substance-predicate logic of Aristotle, in which relations are "driven underground," to remain implicit and unacknowledged. Thus we get no *Principia Mathematica* from Aristotle, and we don't get one until relations/relata are granted their pre-eminent role by Russell and Whitehead.

Let's also consider the case of a lone, notational mark such as "p." I choose the letter "p" because it is often used by logicians to stand for a statement that can be true or false. Again we have a usage in which relations are kept implicit. The letter "p" can only be a statement if it can be "unpacked" into a more explicit statement such that at least one relation and two relata are evident. If "p" cannot be so unpacked, it can only be a *name*, and a name falls short of making a statement. To be sure, names are prerequisite to the making of statements. Once we name a relation and at least two relata, we can then juxtapose those names to make a statement. We thereby build, out of notational marks, a model of the factual situation we mean to refer to. The ontological situation—the "fact"—referenced by the statement, must itself break down into relation and relata components, if this formal "logic of relations," by its own criterion, is to make any successful reference. That criterion pertains to "common relational structure" between a statement and its referent. "Structure" is defined in terms of relations and relata.

We're on our way to defining mathematical structure in its full generality, but at this point, we shall "rein in" and narrow our scope. We shall restrict ourselves to finite structure, so as not to rely on infinities in any way. We proceed by "starting at the bottom," defining the very simplest finite structure, and then, by finite combinatorics, we systematically "climb the ladder" to any desired degree of complexity, defining each and every structural possibility along the way.

An unpackable name refers to "a logical simple," which has no further breakdown into other relations or relata, and hence no structure. If we wanted a "zero" at the bottom of our ladder of structures, in analogy to arithmetic, we could coin the term "null structure," which we might then apply to the unorganized multiplicities called "sets." The next simplest structure after "null" is referenced by the simplest possible statement, R(a,b).

Consider these two statements: R(a,b); R(b,a). The left-right order of the arguments either makes a difference in what is stated, or it doesn't. If we wish it to be irrelevant, we ignore left-right order in the expression and call R a "symmetrical relation," such that R(a,b) and R(b,a) are equivalent expressions. This is where we narrow our scope again. We shall confine subsequent exploration to a domain in which the left-right order of arguments *does* make a difference, such that R(a,b) and R(b,a) are two distinct statements. We shall also shift our reliance to *juxtaposition* in order to indicate the relation in our expressions, dropping the "R" to reduce clutter. We're left with *ordered-pair notation*. Now we can state two axioms:

1. For any a, not (a,a). (No individual shall be related to itself by the ordered-pair relation.)
2. For any a and b, not both (a,b) and (b,a). (The ordered-pair relation shall admit no such instance of symmetry.)

We hereby adopt those two axioms, and there is one more yet to come. The effect of this is to further narrow the domain of mathematics that we are exploring. With a third axiom, we shall come to rest in the specific domain I am interested in.

Next, we consider a particular *finite field* for our ordered-pair relation, one that has exactly 3 individuals: a, b, and c. By using all combinations, we can form six distinct primitive statements:

(a,b) (a,c) (b,c) (c,b) (c,a) (b,a)

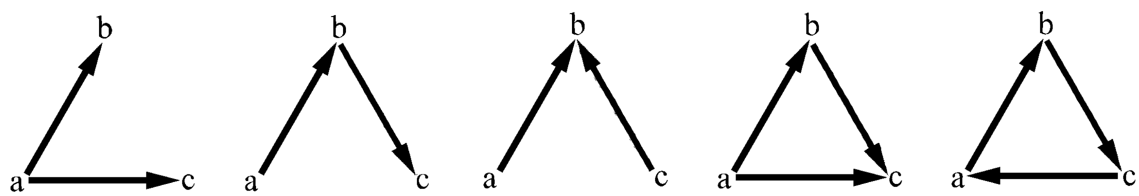
Any of the six expressions, taken alone, is well-formed and legitimate. We have avoided transgression of the first axiom. But each of the six expressions is incompatible with one of the others, by the second axiom. We shall base further development on the permissible logical conjunctions of primitive statements, so let us form a complete list of every distinct conjunction of the primitive statements that does *not* transgress upon the second axiom.

1. (a,b) and (a,c).
2. (a,b) and (b,c).
3. (a,b) and (c,b).
4. (a,b) and (c,a).
5. (a,c) and (b,c).
6. (a,c) and (c,b).

7. (a,c) and (b,a).
8. (b,c) and (c,a).
9. (b,c) and (b,a).
10. (c,b) and (c,a).
11. (c,b) and (b,a).
12. (c,a) and (b,a).
13. (a,b) and (a,c) and (b,c).
14. (a,b) and (a,c) and (c,b).
15. (a,b) and (b,c) and (c,a).
16. (a,b) and (c,b) and (c,a).
17. (a,c) and (b,c) and (b,a).
18. (a,c) and (c,b) and (b,a).
19. (b,c) and (c,a) and (b,a).
20. (c,b) and (c,a) and (b,a).

There are 20 legitimate conjunctions in all. Let's diagram them.

1. (a,b) and (a,c) 2. (a,b) and (b,c) 3. (a,b) and (c,b) 13. (a,b) and (a,c) and (b,c) 15….



fork series convergence triangle ring-triangle

I've diagrammed only five of the twenty statements because five are sufficient to represent all twenty, in a sense that will presently be made clear. (I've also provided an informal name for each diagram, placed along the bottom.) Compare each diagram with the statement above it, until you grasp this method of "diagramming sentences."

Each primitive fact is the linking of two individuals into an ordered pairing. Primitive facts, in turn, are linked together by having an argument in common. For example, in statement number 1, "a" is the predecessor-argument of both ordered pairs. Thus, the ordered pairs link the individuals, and the individuals link the resulting ordered pairs. The outcome is a finite number of structural formations.

What is the link between a statement and its diagram? It is the one-to-one correspondence that matches each labeled arrow of a diagram to one of the ordered pairs in the statement, and vice versa. The direction of an arrow, from one labeled endpoint to the other, matches the left-to-right order of arguments in the correlated ordered pair.

A person who grasps the above method of diagramming formal statements has mastered the use of *isomorphism*, even if the word "isomorphism" be unknown or unfamiliar. The same goes for a person who can use a street map to navigate a city. An arrow diagram is a "map" of its corresponding statement. Because the mapping is 1-to-1 and "goes both ways," we may also consider the ordered-pair statement to be a map of its diagram. The fact that the arrow notation is "pictorial" does not prevent it from being a formal system in its own right. As it happens, I first became interested in the arrow notation, which is more intuitive to me, and only later concocted the ordered-pair notation as an alternative descriptor of the same facts.

Let us "freeze" all formal development in its current state, and make do with ordinary language from here on in. What remains to be said can be clarified by referring to the formal expressions that have already been obtained, which constitute the "bedrock of fact" in a world of exactly three individuals. What we do for a "world of three" can be done for a world of four, five, six, or any finite number "n."

Suppose we swap two labels on the *fork* diagram, putting "a" at the top and "b" at the bottom-left. The diagram then translates to statement 9 in the list of 20. If we then swap "b" and "c," we match up to statement 10. So each of the statements 1, 9, and 10 maps to the *fork*. Likewise, statements 3, 5, and 12 map to the *convergence* The swapping of labels on a diagram corresponds, on the statement side, to a role reversal of individuals in the structure of a fact. The *series* accounts for six of the 20 statements: 2, 4, 6, 7, 8, 11. That's because each node of the *series* diagram has a unique relative position in the structure, so there are six distinct ways to assign three labels to the nodes. In the case of *fork* or *convergence*, only the node where the arrows join has unique relative position; the other two nodes have indistinguishable "map directions;" for example, in the case of statement 1, the directions to either "b" or "c" are the same: Take one step away from "a." Thus there are only three labelings of the fork that count as distinct: one with "a" at the anchor position, one with "b," and one with "c." And finally, among the longer statements, 15 and 18 are *ring-triangle* and the remaining six are *triangle*.

The degree of visual symmetry that we detect in a given diagram is captured in the formalism by the number of isomorphisms, among the 20 statements, that map to the given diagram. I need to clarify the meaning of "isomorphisms among the 20 statements." Statements that map to a given diagram also map to one another. For example, statements isomorphic to the *fork* are isomorphic to each other as well. The 20 statements partition into 5 subsets according to isomorphism: *ring-triangle* maps to 2 isomorphic statements; *fork* and *convergence* map to 3 statements each; *series* and *triangle* map to 6 statements each. That accounts for all 20 statements. Those subset cardinalities—2, 3, 6—seem to rank the five diagrams as to their symmetry: *ring-triangle* most symmetrical; *fork* and *convergence* less symmetrical; *series* and *triangle* least symmetrical. That ranking is in fair agreement, I think, with intuitive ranking of symmetry based purely on visual appearance of the diagrams as drawn. It seems that our formal system is capable of providing a mathematical definition of "symmetry" and its measure.

Among the meager possibilities of structure in a domain of three individuals, the *ring-triangle* is perhaps the loveliest and the most interesting, so it might come as a disappointment to find that the "third axiom" I've been promising is expressly intended to "weed out" the *ring-triangle* from the universe of discourse, leaving a set of possible structures that is poorer by one. As an axiom for arrow diagram construction, we can just say, "Don't draw the *ring-triangle*." For the ordered-pair notation, we could just negate statements 15 and 18 and transplant them from the list of facts to the list of axioms. That's a "dirty way" to accomplish the intended result, but it works, on a case-by-case basis. If we were to consider the next higher domain—the world of four individuals—there would be many *ring-triangles* to weed out; we won't allow them to occur even as substructures of some larger structure. There would be many "ring-quadrilaterals" also, and we intend to exclude all ring-like formations from the domain of possibilities. To cover all cases, we could proceed as follows: define "serial path" (intuitively, any traceable path within an arrow diagram that obeys the direction of the arrows); define "end point" (any individual of a serial path that appears only as a predecessor and not as a successor, or vice versa); and finally, stipulate that all serial paths must have an end point. That will weed out all ring formations, because a ring is a serial path with no end point.

Why would we exclude "ring-statements" from the well-formed formulas in our formal system? This has to do with time not going backward, and with causal anomalies, such as an individual which is its own causal ancestor and descendant. The exclusion of ring-statements will customize the formal language for strict isomorphism of its statements to a particular and important domain of reference, which Whitehead calls "temporal succession," and Russell calls "causal structure." The 1-to-1 mapping of formal statements to the facts of temporal succession is effected by interpreting each primitive statement (each arrow of a diagram) as a step in time from one individual moment to another. I do not believe that either Whitehead or Russell, in developing *eventism*, ever gave full attention, as this paper does, to the "finite case." Situated at the "nursery level" of development are the simplest and clearest facts that the theory can express. Before we become adults and set our thoughts on infinity, let's make sure we understand our abc's.

"So time actually forks?" (Yes Johnny.)

"And given enough moments, time could make just about any pattern out of series, fork, and convergence?" (That's right Johnny.)

"And time never goes backward?" (No Johnny, it doesn't.)

"And the idea of a finite amount of time makes sense all by itself?" (Well Johnny, I hope it does, because the notion of infinity is really no help at all in understanding time.)

"What about space-time, particles and waves, energy, and all those numbers? Is all that stuff just funny patterns of time sequence?" (Very good, Johnny—that's exactly right.)

The ring-exclusion axiom, as formulated above, in terms of "serial path" and "end point," does the work of the first two axioms as well. Axiom 1 precludes a ring path of one step from an individual to itself. Axiom 2 precludes a ring path with two steps and two individuals. This makes the ring-exclusion axiom seem less "quirky," and more like a natural extension of the first two axioms. Nevertheless, the effect of Axiom 3 is to sacrifice flexibility of expression in the interests of trimming the language for tighter correspondence to a chosen domain of reference. That chosen domain is physics, as seen from the vantage point of *eventism*. Axiom 3 is a formal implementation of what Stephen Hawking calls "chronology protection." Some time ago, Kurt Godel showed that General Relativity, as currently formulated, admits of various solutions in which temporal progression forms a closed loop. We see the same thing, in finite form, in the *ring-triangle* Hawking is urging the implementation of chronology protection in physical theory, but many physicists are resistant to that, and would rather have the flexibility to employ "cyclic time" and "reversible time" in their theoretical conceptions. To treat these competing views formally, we could revert to a version of the formalism (call it "Version 1") that incorporates Axiom 1 *only*, in order to accommodate reversible time; we can stipulate "Version 2" to include Axioms 1 and 2, which allows cyclic time but not reversible time; and our latest axiom can define "Version 3," which implements "chronology protection."

We have other domains of reference besides physics, for which we need the wider scope of reference afforded by either Version 1 or 2. My primary topic is the adequacy of language for *any* finite domain—not just the domain of physical theory. So let's step back from specific domains of reference and address the primary topic. I have set up the formalism as an "ideal language," in the manner of a school of philosophy which relies on such a technique. (The one who influenced me in this regard was Gustav Bergmann.) The technique of "ideal language" goes beyond the logician's concerns with formal language, and beyond the mathematician's concerns, extending to the ontological implications of formal language for the expression of contingent matters of fact. Furthermore, I have laid out the basic construction by assembling individuals into "primitive facts" (or "atomic facts") and these in turn are assembled into "molecular facts" by means of the conjunction "and." This is very much in line with Wittgenstein's *Tractatus*, which Russell embraced with enthusiasm, calling it "logical atomism." Russell adopted this label for his own mode of thought, and never gave it up.

From the standpoint of pure formal logic and mathematics, our construction of permissible statements is algorithmic and unassailable: start with any number—say 3—and generate 3 names; generate all ordered pairs of those names; generate all combinations of those ordered pairs; comb through the combinations, checking against the axioms, and delete any combination that fails any axiom; print out the list of combinations that remain. This printout—the output of the algorithm—is the *primary language* of our formalism. It is free of negation and variables. It is free of axioms. It is "kept clean" in order to match up 1-to-1 with the limited alternatives of fact that could obtain in a domain of reference involving three individuals. The axioms, even though they are needed to generate "the facts," are designed with the output in mind, and are best treated as a meta-language that makes reference to formal features of the primary language. There is "standard logic" for formalizing the use of variables, negation, and the functions of "all" and "some." This standard logic is like truth tables for the finite combinations generated in our primary language, so it is standardized and agreeable to all. My axioms have been given only pseudo-formal expression, but I think they are clear enough for our purposes.

What makes our constructions markedly different from the *Tractatus* is that we propose specific form for the primitive facts, building each one from a pair of individuals and what I would like to call a "contingent" relation. The reason I call it "contingent" is that it can either hold or not hold for a given pair of individuals. It's not like the relations in set theory, which are "geared for tautology," one might say. For example, suppose we take an interest in the *unordered* pair as our primitive fact, and we defined it as a "subset" of the *n* individuals of the world. Our arrow diagrams will lose their arrowheads, since there is no directionality needed to map an unordered pair. We will have diagrams with line segments rather than arrows. The structures of world-3 will be reduced to an "elbow" and a triangle. All that's fine. But in the set terminology of the facts, the "elbow" depends for its form on a *missing subset* of the three individuals of 3-world. Subset formation isn't usually considered a hit-or-miss affair—when you *have the set*, you *have all its subsets*. It grates against the very notion of "set" to state contingent facts in terms of subsets that "don't obtain." That's why the reliance on *relation* and *relata* as the primitives of our language is more advantageous than constructing everything from set theory. We want a language that can state a simple contingent fact in a simple intuitive manner. The "language of relations" works for that purpose, I think. That is why I suggest that the referential use of the ordered-pair is generally geared for what we might call "contingent relations."

Let me discuss the "design considerations" that have resulted in a set of statements (26 in the world-of-3) that I am calling "the primary language" and the "bedrock of fact." The driving goal is to demonstrate the adequacy of language to the task of stating any fact that could hold true in a domain of *n* individuals that constitute the field of a pair-wise relation. (I intend to show that 2-termed relations are fully general and make higher-termed relations redundant. Furthermore, in a domain for which *more than one* 2-termed relation is defined, the same structural possibilities are regurgitated for each additional relation, so nothing new or problematic comes about. Thus, the minimal case of a single pairing relation is a paradigm case of full generality.) The formal language required to state all the possible facts in some finite domain of reference, should itself be finite. This consideration enters in again and again in the design. Language tends to "run away with itself" because of the meta-linguistic layers that pile up once a base of primitive expressions is established. Even if that primitive base is finite, as is the case with our *primary language*, an unlimited superstructure of general statements and logic manipulation can "grow off the base." That sort of "runaway" might suggest to someone that language is inadequate to "say everything" about the finite domain of reference. But the accumulating logical superstructure is just "saying in different words" what has already been said *once*, and said *completely*, in the *primary language*. For instance, a general statement about a structure, or about its substructures, or its individuals or its relations, is only considered valid when it logically implies nothing but particular facts to be found in their raw form in the base layer. That's all that can justify the general statement. In that sense, general statements are *meant* to be redundant with some set of particular statements.

Our *primary language*—our base layer of particular facts—is like a "bitmap image" of the possibilities in the domain of reference. It is complete and raw, without having been summarized or sliced-and-diced according to its inherent uniformities or patterns. It is specifically the *primary language* that proves adequate to its finite domain of reference. Consider the logical conjunction "*p and p*." This is usually considered equivalent to "*p*." But I don't even allow "*p and p*" as a well-formed expression among my conjunctive statements. Why not? Because there would be no end to the forming of conjunctions in that case. It would be another case of runaway language. So my conjunctive "and" relation, if I formalized it, would have its own version of Axiom 1—its own formation rule of "irreflexivity" to obey. Also, why doesn't the "null structure" appear in the list of primary statements? How do we say "there is no structure" when none of the individuals have a pairwise relation to one another? For that case, we just don't say anything at all. For any relational facts that obtain, our primary language has a corresponding statement. When no relational facts obtain, our primary language meets the challenge with silence. A perfectly *unordered* set of *n* individuals does not provide any primitive facts, nor does the set itself constitute a fact to be stated.

Suppose for the moment that the formal language I've constructed has been proven adequate to its limited domain of reference by the 1-one-1 mapping that obtains between its statements and the possible facts of its domain. I mean to infer from this that the English language, in its declarative mode, is also adequate for any finite domain of reference. What considerations are necessary to justify that inference? All we need to do in order to prove adequacy, it seems to me, is to show that we can use the English language to say what the formal language can say. If we can do that, it's beside the point to survey the logical pitfalls and pathological expressions that can also arise in English. Therefore the task is trivial, because we can *transcribe* any formal statement into English. Let us transcribe Statement 1 into English:

Formal statement: (a,b) and (a,c)

English sentence: Ay comma be and ay comma see.

That wasn't so hard, was it? Spelling, I think, is not a big concern. My point is that the formal statement already reads aloud like an English sentence, and for all intents and purposes, the formal statement *is* already an English sentence. Only the rudiments of English are needed to express a primitive fact, and only the word "and" is needed to string the primitive facts together. All that's needed in order to preserve the isomorphism of statement-to-fact is to make sure we transcribe distinct names of the formalism to distinct coinages in English, while parsing the words, phrases, and sentences in accord with the clumpings and spacings of expressions in the formal notation.

I adduce the formal language itself then, as proof of the adequacy of English for stating the facts of a finite domain of reference. In that form, the usage is primitive and monotonous, with only particular facts adumbrated. Dull as it is, every phrase holds distinct information, and each sentence holds exhaustive information as to the contingent state of its referent domain. At this point, we have a very skeletal view of English usage for declarative purposes, which has the virtue of stating a precise fact completely. The structural feature required for this success in representational language is isomorphism between the sentence structure and the structure of the fact that obtains in the domain of reference. We have this isomorphism because, by hypothesis, the domain of reference is finite, and no matter what the particular domain is, its finite number of individuals, in their finite number of relationships, entails a finite number of alternative, fully specified possibilities. The formal language, in principle, calculates *all* the possibilities for any finite *n* we might require. It stores these as strings that transcribe straight to English. Or back again. Now we have three formal systems: the pair notation, the arrow diagrams, and skeletal English. They can serve as reference domains for one another, if each is considered a language in its own right.

It is evident that I am treating "language" as a structured object, neglecting to address the functional role of the person who *uses* language. Without this, a "language" is no more than a species of structure which has the *potential* to be employed as a system of reference for other domains of structure. A person must master *naming* in order to acquire the "gift of language" and exploit its potential as a system of didactic reference. And we must start by "getting the idea" of naming individuals. I say this with confidence in light of Helen Keller's account of how she acquired the gift of language, which came late to her because she was deaf and blind. The first name she acquired was a simple tapping pattern that she felt on her wrist. The referent of that name was water, which, at the time, was running over her hand. The teaching efforts had begun with many fruitless sessions in which the teacher's tapping, and the running water, failed to coalesce in Helen's mind as a conjugate pairing of name-and-referent. But eventually, and most suddenly, it happened—and Helen had a name for water. And once the first name was acquired, Helen could repeat the trick at will, and she knew it. She acquired names voraciously, and her vocabulary erupted. She became a writer.

I cannot give an account of the "state of mind" in which name-and-referent are paired in the crucial manner required for the gift of language. I do believe that the intentionality peculiar to "mind" is at the core of the situation. That core is not engendered in a vacuum, but rather in a brain, in a body, in social interaction and communication with others. Thus it sprawls into a vast subject matter. Nevertheless, we all "get" the epiphany that came to Helen Keller if we have acquired any language competency whatsoever. And then, just as naturally, and just as difficult to explain, we learn to link words together in order to indicate a link between their referents.

For Russell, logical atomism is not just *a* mode of thought—it is *the* mode he adopted. He committed himself wholly to it, and fashioned every thought in its mold. That mold compels him to frame any thought within a "master partition" of relation-relata. That master partition is for him the "first distinction"—the "Logos." Our formal developments are constructed from the kernel of that "first distinction," so that everything revolves around relations-and-relata, relational structures, and isomorphisms. If language works by those principles, inadequacies of language to its task must be failings of the user, whose role is to fit names to a domain. That's the whole art of didactic expression, because it requires clear discernment of the structure of the domain to be addressed. Clarity of expression is coupled to clarity of discernment of the subject matter. If we can achieve one, we can achieve both. Language will be adequate. But clear discernment of a domain of reference is what we call "insight" or "understanding," which comes at best in fits and starts. Thus, arriving at a suitable naming scheme for a contingent domain is tantamount to achieving clear insight into that domain, which is not algorithmic or trivial.

Let us return to formal matters, and in particular, to the hypothesis that all relations are reducible to dyadic (or 2-termed) relations. Consider again the five arrow diagrams we have produced. I have called them "structures" and given them common names. Can we think of them instead as "3-termed relations?" Each one relates 3 individuals to one another in a fully specified manner; if we want to define 3-termed relations, we can do no less. And, it seems to me, we can do no more. I think that, for mathematical purposes, it is purely a matter of convenience whether or not to employ more than two terms in the notation for some particular relation. (I am excluding from consideration a non-finite number of terms.) What sort of "convenience" do I have in mind for the formal system, that I choose to build up everything from dyadic relations? It allows us to *index* the stages of development (world-of-*n*) so that thorough accumulating coverage of *all* possibilities of structure, up to domain *n*, is catalogued. In the domain of 10 individuals, *all* 10-termed relations are defined, without regard to their simplicity or eccentricity. If we "carve out a domain," such as "space-time geometry," for which we feel that 4-termed relations will be convenient, such as R(x, y, z, t), we assume that our domain has a highly repetitive and uniform structure that justifies the choice of notation. But now we've filtered out a lot of good possibilities which the theory might have suggested but for the choice of convenient notation. I believe that Russell and Whitehead's *eventism* was hampered in just this way, by an exaggerated uniformity imputed to space-time relations.

Next I want to define a "molar relation," and argue that such a relation is a construction that can and should be avoided. Our "layer of fact" is "flat," in the sense that only individuals enter into the contingent pairwise relation. What if we allow a primitive fact to have a contingent relation to an individual? We get a statement such as (a, (b,c)). If we diagram this, we get a familiar arrow for the "nested fact" labeled "b" at one end and "c" at the other. To complete the diagram we need a second arrow labeled "a" at one end, but the other end must connect to the first arrow in a new way, without terminating at either "b" or "c," and without forming a new junction that would imply a fourth individual. I contend that what we want to say with the notation for the molar fact can be said as follows: (a,b) and (a,c) and (b,c). The molar fact is "distributed out."

There are still molar facts in our expressions however. Our "molecular facts" are molar in the sense that primitive facts enter into the *conjunctive-and* relation with one another. That is, our "layer of fact" is not "flat" as I had claimed. It has two tiers of construction, making use of two distinct relations. The "and" relation seems weak, as though it doesn't really say anything important. And indeed, it is redundant! I only realized this a couple pages ago, even though I had implied it a few pages before that. I quote myself: "Thus, the ordered pairs link the individuals, and the individuals link the resulting ordered pairs." The use of "and" to assemble primitive facts into longer strings, and the use of separate lines to keep the longer strings distinguished from one another, are redundant niceties. To see this, consider the following. If we destroy the formatting, leaving only an unordered set of primitive facts, we lose no information, and we can relink the facts at will, according to the individuals that populate more than one fact. We also have each primitive fact represented more than once, but never twice in the same molecular statement. This "never twice" condition suffices, for the world-of-3, to keep the 26 statements "unlinked" and "put onto separate lines." The primitive facts have their own assembly information coded within. They *assemble themselves* into all their structural possibilities. The formatting just emphasizes this in a manner that is easier for us to discern. All we need for our "layer of fact" is an unordered set of 54 primitive statements (the total number of primitive expressions in the 26 statements.) Our algorithm for Domain 3 can do its calculations and just print out 54 primitive statements in no particular order. All the information is there. Our "layer of fact" is truly "flat" after all, and we need employ no "molar relations."

Why does the word "and" insinuate itself so forcefully into our statements? It is not needed for the arrow diagrams, where no individual appears more than once per diagram. It has to do with the *seriality* of certain languages, and English is such a language. As we map an arrow diagram into ordered-pair notation, we are mapping a non-serial language to a serial one. For that we need "topological magic," which requires some kind of "fold-over." We "chop up" the diagram into its individual arrows. In the case of "fork," we make the "chop" at the anchor point, which chops that individual in two. That individual then appears as a *repeated name* in the ordered-pair notation for the fork. The word "and" just emphasizes the link represented by the dual instance of one individual name. Therefore, name repetition, and the word "and," refer to nothing in the domain of reference. They are artifacts of the seriality of the language itself.

Now I can finish the argument that dyadic relations are sufficient for denotation of structure. Again we shall situate ourselves in the World-of-3 as an example. We *might* consider variegating the 3 individuals into "types." Also, we *must* consider that more than one dyadic relation can be defined for the 3 individuals. This latter consideration makes the former one unnecessary, I think. Conferring a "type" on an individual cannot make it "more individual" than it already is. Its *type* could only serve to relate it to other individuals of the *same type* But for that, we call upon a relation that has, as its field, individuals of the same type. That is, if two relations are defined over a single domain of individuals, and if those two relations have distinct fields, then those two relations can be said to "type" the individuals according to the distinct fields. Nothing else is to be gained from "typed individuals." So let us define *two* relations for our domain of three individuals. Each relation may be dyadic, without loss of generality—I think I've proved that part already. Each relation may have a field of 2 individuals or a field of 3, which is all the World-of-3 affords. Let's allow each to have a field of 3, for maximum complexity. Both relations shall obey Axiom 1, because self-relatedness is a frivolous notion that only does harm. To obtain the utmost generality, we do not impose Axiom 2 or 3. We can still use arrow diagrams, but each arrow can have either one arrowhead or two, affording all combinations of asymmetrical pairings and symmetrical pairings. If we diagram all the possibilities of structure for just one of the relations, we get an expanded set of triangular figures similar to the ones we've looked at before. If we then draw a second set of diagrams for the second relation, we get a duplicate set of diagrams. Each distinct structural possiblity in a domain of two relations is then mapped by a *pair* of diagrams, paired by being labeled with the same 3 names. My point is that we can define as many dyadic relations as we like (keeping it finite) over a given domain of individuals, and never outrun the adequacy of language for a finite domain of reference.

As I said earlier, I began to draw arrow diagrams without any foresight that these would have an analog in formal logic. I was in the middle of writing a book, The Mind-Body Problem and Its Solution, and I was into Chapter 5 called "Space-time as Causal Structure." I had no particular "beef" with the notions of "continuity" or "space-time continuum," but I decided to make a concession in that regard, because the ideas I understood best could be depicted with a few discrete arrows, which I could draw. My first diagram was one arrow. Then came the simplest "causal chain," which is the *series* diagram shown earlier in this paper. Then came the *fork*, *convergence*, and *triangle*. I began to think of these simple depictions as "things that time can do." It wasn't long before I noticed something important. The *triangle* consists of two separable paths that have common endpoints; these endpoints mark a single time span that applies equally well to either of the two paths; that single time span is broken into two temporal transitions for the one path, and unbroken for the other path; thus a ratio of 2:1 is formed that is quite naturally called "the relative temporal frequency" of the two paths. I also had it in mind that frequency and energy are tied together at the quantum level by Planck's constant. That led me to pursue a structural definition of "energy" in terms of relative temporal frequencies. With that development, each arrow of a diagram takes on the interpretation of "a quantum of energy."

As soon as I had finished the mind-body book and got it published, I thought it would be worthwhile to extract, for separate publication, the small amount of formal material that was "physics proper." I titled it A Theory of Everything for Physics. When I thought I was about finished, the most unexpected discoveries arose, one on the heels of another, including the simple structure of the neutrino and the electron. These so-called "fundamental particles" actually consist of a repetitive cycle with an arrow diagram of six individuals. I discovered them innocently enough by "climbing the ladder" of finite domains discussed in this paper, and came to a highly symmetrical diagram that could "stencil" a 4-dimensional time lattice.

I've spent a year now trying to interest someone—anyone—in my discoveries, but there is great resistance. It must strike the professional physicist as unbearably simple-minded. Apart from that, it is difficult to conceive that time alone has enough "substance" to account for physics all by itself. I don't think even Russell or Whitehead could quite believe it. But when their eventism is applied to the finite domain, it virtually assembles itself into a stunningly simple solution to physics. A person could stumble upon it accidentally, by this line of questioning: *What is the simplest theory—theory of anything—there could possibly be? Without at least one relation, there are no facts, and there is no theory. In that case, I'll have just one relation. Now what is the simplest of all relations? And what is its minimal field?* The "logical atom" eked out of this miserly development finds its referent as the quantum of energy—the quantum of time—as soon as the referential apparatus is pointed toward the domain of physics.

I have not yet been able to mingle with the caste of physicists directly, but I procured a copy of an excellent paper by Rafael Sorkin (see reference [1]) which corroborates the usefulness and power of our formal system for reference to the domain of physics. What I describe as a fact of temporal succession (under Axiom 3, chronology protection) he calls "a causal set." The theory of causal sets is being applied to such matters as the Hawking radiation at the boundaries of black holes. I assume that Stephen Hawking has the theory of causal sets specifically in mind as the vehicle for incorporating chronology protection into physics. I just reread Mr. Sorkin's paper, finding passages here and there which I could actually understand. It provides another notation called a "causal matrix," which is the "truth table of a fact." Below is the truth table of a fact in our World-of-3, which requires only a 3x3 matrix to say “(a,b) and (b,c) and (c,a).”

	a	b	c
a	x	1	0
b	0	x	1
c	1	0	x

(Oops—I stated a *ring-triangle* fact.) The diagonal is “x’d out” because of Axiom 1. Axiom 2 precludes two 1’s from occupying mirrored positions across the diagonal. With just those axioms, there are 27 ways to fill in the table with 0’s and 1’s. One of those ways is all 0’s, which we have treated as a null structure or set, and not as a fact, so this formalism serves to verify our previous accounting, which gave us 26 possible facts for a World-of-3. The truth table is an excellent format for “splaying” the fact into *bits* of information. Recalling that the ranking of “symmetry” seems also to be numerically definable, the pieces might all be in place for a fully general definition of “complexity” of physical structure, such that each possible fact is assigned a numerical value on a scale of simple-to-complex.

Reference [1]: *The Causal Set as the Deep Structure of Spacetime*, Rafael D. Sorkin, Department of Physics, Syracuse University, Syracuse, NY 13244-1130, U.S.A.